Specimen Paper C

1 Differentiate (a) $y = \dfrac{\ln(x^2+1)}{2x}$ (b) $y = \sqrt{x}\sin^{-1}\sqrt{x}$ **3,3**

2 $z = \cos\theta + i\sin\theta$

Show that $z\bar{z} + \dfrac{z}{\bar{z}}$ can be expressed in the form $p + q\cos 2\theta + r\sin 2\theta$,
Stating the values of p,q and r. **4**

3 Use the substitution $x = t^2$ and integration by parts to find

$\int (1+\sin\sqrt{x})dx$ **6**

4 Find a formula in terms of n for $\sum_{r=1}^{n}(5-2r)$

Hence evaluate $\sum_{r=11}^{30}(5-2r)$ **4**

5 Express in partial fractions $\dfrac{2x-1}{x(x^2+1)}$ **4**

6 A curve is defined by the equation $3x^2 - xy + 5y^2 = 7x$.
Given the point A(1,1) lies on the curve
find $\dfrac{dy}{dx}$ and the equation of the tangent at the point A. **5**

7 Prove by induction $1+\dfrac{5}{2}+4+\ldots+\dfrac{3n-1}{2} = \dfrac{n(3n+1)}{4}$, n a natural number, **4**

8 A geometric series is defined by $5+\dfrac{5x}{(x-1)}+\dfrac{5x^2}{(x-1)^2}+\ldots+\dfrac{5x^{n-1}}{(x-1)^{n-1}}$

Write down the common ratio r of the series and find a formula
for the sum of the series to n terms in its simplest form.

Verify the formula works for the sum to 2 terms. **5**

9/over

9 Find an equation of the plane ***P*** which passes through the point (3,5,-1) with normal parallel to $i + 2j - 3k$.

Find the point of intersection of the line $\dfrac{x+2}{4} = \dfrac{y-2}{3} = z$

and the plane ***P*.** **2,3**

10 A recurrence relation is defined by the formula $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{11}{x_n}\right)$

Given $x_0 = 3$ calculate x_1, x_2 and x_3 to 3 significant figures.

Find the fixed points of this recurrence relation. **4**

11 Find the Maclaurin series for $\log_e(1+x)$ up to terms in x^3.

Hence find the Maclaurin series up to terms in x^3 for $\log_e(1-2x)$ **3,2**

12 Find the matrix **A** associated with reflection in the *y*-axis

and the matrix **B** associated with an anti-clockwise rotation of $\dfrac{\pi}{4}$

Find the matrix **AB** and find the image of the point (*x*,*y*) under the transformation matrix **AB**.
Hence write down the coordinates of the image of (2,0) under this transformation. **5**

13 Prove that that the following statements are true or false, if false provide a counter example, where *n* is any natural number.

(a) $n^4 - n^2$ is always even, n any natural number >1.

(b) $n^4 + 1$ is always a prime number. **3**

14 **(a)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 50e^{-x}$$

(b) Find the particular solution given at $x = 0$, $y = 0$ and $\dfrac{dy}{dx} = 0$ **7,3**

15/over

15 A class of 15 Advanced Higher students are given the golden opportunity of attending extra classes during a very sunny Easter Holiday.

The long suffering teacher conjectures that the number of students who attend satisfies a differential equation $\frac{dP}{dt} = k(15 - P)$, P is the number of students, t is the number of days.

(a) Given at t=0, $P = 0$ show that $\frac{1}{15 - P} = Ae^{kt}$, stating the exact value of A.

Hence find a formula for P explicitly in terms of t.

(b) After 3 days 6 students attend, find the value of k to 2 significant figures.

(c) On the 4th day the weather changes, does this affect the number of students ?

(d) As the exam draws closer more and more students arrive, how many days does it take 10 students to attend ?

5,2,2,1

16 A function f is defined by the formula $f(x) = \frac{x^2}{(1 - x^2)}$

(a) Write down the equations of all 3 asymptotes. **3**

(b) Show that f has only one stationary point.
Find the coordinates of the point and justify its nature. **4**

(c) Sketch the graph of $y = f(x)$ showing clearly what happens as $x \to \pm\infty$ **2**

(d) On the same diagram sketch the graph of $y = |f(x)|$ **2**

17 A complex number is defined as $z = \cos\theta + i\sin\theta$
Write down an expression for z^4 using the binomial theorem and another expression using de Moivre’s Theorem. **3**

Hence equating real parts write down an expression for $\cos 4\theta$ in terms of $\cos\theta$ and $\sin\theta$. **3**

Express $\cos 4\theta$ entirely in terms of $\sin\theta$. **3**

End of question paper **Total 100**